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INDUCED GRAVITATION AS NONLINEAR ELECTRODYNAMICS EFFECT

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Abstract

The effect of induced Riemann geometry in nonlinear electrodynamics is considered. The possibility for description of real gravitation by this effect is discussed.

1 Introduction

The idea about a possibility for description of gravitation as manifestation of another interactions was said by Sakharov[14)] in context of quantum field theory. Adler[1)] proposed to consider the gravitation as manifestation of electromagnetic interaction again in the scope of quantum field technique. Such view to gravitation is in the direction of unified approach for all interactions of material objects.

But in classical nonlinear electrodynamics, in particular, in Born-Infeld model there is an effect which can be considered as means for description of gravitation. This effect can be named as induced Riemann geometry in nonlinear electrodynamics. It appear when we consider the problem on movement of electromagnetic soliton or propagation of light with some given field, for example, of distant solitons. This problem can be considered as initial part of iterative procedure for obtaining complex solution including many solitons and radiation field. The given field modifies the trajectories of soliton and light beam. This modification can appear as effective Riemann space for the movement of solitons and the propagation of light, with metric depending on the given electromagnetic field components.

For the first time, seemingly, the effective Riemann space appearing in nonlinear electrodynamics was discovered by Plebansky[13)] in the problem on propagation of light in given field for Born-Infeld model. Effective Riemann space in the problem for movement of electromagnetic soliton in given field was discovered by Chernitskii[3)] for Mie nonlinear electrodynamics and for Born-Infeld type models (Chernitskii[4)]).

At the present time Born-Infeld electrodynamics (Born & Infeld[2)]) arouses considerable interest from various points of view. In particular, it appear also in quantized strings theory (Fradkin & Tseytlin[10)]). Here we shall consider just Born-Infeld nonlinear electrodynamics model.

2 Born-Infeld electrodynamics

The Born-Infeld electrodynamics is derived from a variational principle proposed by Eddington[9)]. He also had in view some unified approach. This principle has the following very geometrical form:

$$\delta \int \sqrt{|\det\left(g_{\mu\nu} + \chi\, F_{\mu\nu}\right)|}\,(\mathrm{d}x)^4 = 0 \quad , \tag{1}$$

where the Greek indices take values $0, 1, 2, 3$, $g_{\mu\nu}$ are components of metric for space-time, $F_{\mu\nu}$ are components of the tensor of electromagnetic field, χ is a dimensional constant.

3 Effective Riemann space

The Born-Infeld system of equations (see also Chernitskii[5)]) has the following very notable form of the characteristic equation (Chernitskii[6)]):

$$\mathfrak{g}^{\mu\nu}\,\frac{\partial\Phi}{\partial x^\mu}\,\frac{\partial\Phi}{\partial x^\nu} = 0 \quad , \tag{2}$$

$$\mathfrak{g}^{\mu\nu} \equiv g^{\mu\nu} - \chi^2\, T^{\mu\nu} \quad , \tag{3}$$

where $T^{\mu\nu}$ are components of symmetrical energy-momentum tensor, $\Phi(x) = 0$ is a characteristic surface.

It should be noted that form (2) of the characteristic equation is typical for a broad class enough of nonlinear electrodynamics models (see Glinskii[11)], Novello et al[12)]). But for the Born-Infeld electrodynamics the tensor $\mathfrak{g}^{\mu\nu}$ includes (3) the energy-momentum tensor of the model!

As it is known, if we have some given field $\overset{g}{F}_{\sigma\rho}$, the problem for propagation of quick-oscillating wave with small amplitude gives the dispersion relation coinciding with the characteristic equation (2) where $\Phi(x)$ is wave phase and $\mathfrak{g}^{\mu\nu} = \mathfrak{g}^{\mu\nu}(\overset{g}{F}_{\sigma\rho})$. Thus the light propagates in the effective Riemann space induced by the given electromagnetic field.

The matter is constructed by charged particles which are in permanent movement. Thus a massive body can look as a very complicated non-stationary electromagnetic field configuration. We can use Fourier expansion for this field configuration in proper coordinate system $\{y^\mu\}$ of the body. We assume here that in the proper coordinate system this field configuration is time-periodical but this is not essential and the period can rush to infinity. Thus for moving body in the coordinate system $\{x^\nu\}$ we can write the following:

$$\mathcal{D} = \sum_{f=-\infty}^{\infty} \mathcal{D}_f \,\exp\left(\imath f \Theta\right) \quad , \tag{4}$$

where $\mathcal{D}$ is the column with six components of the electromagnetic field, $\mathcal{D}_f = \mathcal{D}_f(y^i)$,
the Latin indices (except f) take values $1, 2, 3$,
$y^i = L^i_j\,[x^j - a^j(x^0)] \quad , \qquad L^\mu_\nu$ are component of Lorentz transformation matrix,
$a^j = V^j\,x^0 + a^j_0$ is the rectilinear trajectory,
$\underline{\omega}\,y^0 \equiv \Theta = \Theta(x) \quad , \qquad \dfrac{\partial\Theta}{\partial x^\mu} \equiv k_\mu \quad ,$ and

$$|g^{\mu\nu}\,k_\mu\,k_\nu| = \underline{\omega}^2 \quad . \tag{5}$$

The static field configuration $\mathcal{D}_0$ in (4) is the aggregate of distributed charges such that the full charge equals zero. The movement of charged electromagnetic solitons in small given field $\overset{g}{F}_{\sigma\rho}$ was considered by Chernitskii[4)] for Born-Infeld type electrodynamical models. Here the given field is small with respect to the maximum value for the field of the soliton under consideration. Using an approximate method we obtain the trajectory $a^j(x^0)$ modified by the small given field. The first order by the small field $\overset{g}{F}_{\sigma\rho}$ gives the Lorentz force (see also Chernitskii[7)]). The second order gives the trajectory in the form of geodesic line equation for

some effective Riemann space with metric depending on squares of the field components $\overset{g}{F}_{\sigma\rho}$. Thus sign of charge does not matter for this effect.

The quick-oscillating part of the field configuration (4) (for $f \neq 0$) is some standing wave in proper coordinate system. The simplest example for such standing wave is the function

$$\frac{\sin(\underline{\omega}\, r)}{\underline{\omega}\, r}\, \sin(\underline{\omega}\, y^0) \quad , \qquad r = \sqrt{y^i\, y_i} \quad , \tag{6}$$

which is the solution for wave equation in spherical coordinate system. If we operate on this function by the Lorentz transformation or consider it in the coordinate system $\{x^\mu\}$ we obtain the prototype for the quick-oscillating part of (4).

The movement of such semi-standing wave having a small amplitude was considered by Chernitskii[7]. The additional given field $\overset{g}{F}_{\sigma\rho}$ modifies the originally rectilinear trajectory of the semi-standing wave such that the dispersion relation (5) is modified to the following:

$$|\mathfrak{g}^{\mu\nu}\, k_\mu\, k_\nu| = \underline{\omega}^2 \quad , \tag{7}$$

where $\mathfrak{g}^{\mu\nu}$ are defined by (3) with $\mathfrak{g}^{\mu\nu} = \mathfrak{g}^{\mu\nu}(\overset{g}{F}_{\sigma\rho})$.

Relation (7) is coincide with the Hamilton-Jacobi equation for a massive particle moving in gravitational field. For the case $\underline{\omega} = 0$ we have the dispersion relation for light.

The stated results argue for that the massive bodies, like the light, will be moving into the effective Riemann space induced by the electromagnetic field of distant bodies.

4 Gravitation

Let us write the following zero speed approximation for the geodesic line equation of the effective Riemann space:

$$\frac{\mathrm{d}V_i}{\mathrm{d}x^0} = -\,\frac{1}{2}\,\frac{\partial \mathfrak{g}_{00}}{\partial x^i} \quad . \tag{8}$$

For sufficiently small given field we can write

$$\mathfrak{g}_{00} = 1 - 2\,\epsilon \quad , \qquad \text{where} \qquad \epsilon \equiv \frac{\chi^2}{4}\left(\overset{g}{\mathbf{E}}{}^2 + \overset{g}{\mathbf{B}}{}^2\right) \quad . \tag{9}$$

Let us define the Newtonian potential φ as averaging of ϵ on a small four-dimensional volume δX^4 for removal of quick-oscillating part. Thus we have

$$\varphi = -\,\frac{1}{\delta X^4}\int\limits_{\delta X^4} \epsilon\; (\mathrm{d}x)^4 \quad . \tag{10}$$

On the other hand in Newtonian gravitational theory we have

$$\varphi = -\,G\,\frac{M}{r} \quad . \tag{11}$$

Let us define the given field as the sum of two quick-oscillating parts: field of massive neutral body at essentially distant region from the body $\overset{\circ}{F}_{\mu\nu}$ and some background radiation $\overset{\approx}{F}_{\mu\nu}$. We can write the following relations:

$$\overset{\circ}{F}_{\mu\nu} \;\sim\; \frac{\sin(\underline{\omega}\, r + \overset{\circ}{\phi}_r)}{\underline{\omega}\, r}\, \sin(\underline{\omega}\, t + \overset{\circ}{\phi}_t) \quad , \qquad \overset{\approx}{F}_{\mu\nu} \;\sim\; \sin(\mathbf{k}\cdot\mathbf{x} + \overset{\approx}{\phi}_r)\, \sin(\underline{\omega}\, t + \overset{\approx}{\phi}_t) \quad . \tag{12}$$

It should be noted that there is an interconsistency between the phases of the fields $\overset{\circ}{F}_{\mu\nu}$ and $\overset{\approx}{F}_{\mu\nu}$ because of interaction. Thus from (10), (9), (12) we can write the following evaluative expression:

$$\varphi = -\frac{\chi^2}{2}\left(\overset{\circ}{G}\, M\,\frac{1}{r}\right)\overset{\approx}{G} + O\left(\frac{1}{r^2}\right) \quad . \tag{13}$$

Here $\overset{\circ}{G}\, M/r$ characterizes the field of the body $\overset{\circ}{F}_{\mu\nu}$, where we assume that it is proportional to the mass M. And $\overset{\approx}{G}$ characterizes the background field. Thus we have

$$G = \frac{\chi^2}{2}\overset{\circ}{G}\ \overset{\approx}{G} \quad . \tag{14}$$

The model constant χ can be obtained in electrodynamical experiments (see, for example, Denisov[8)]).

Most likely we can take $\overset{\circ}{G} = \text{const}$. But the background field can has weak space dependence, for example, on density of distribution for the massive bodies and $\overset{\approx}{G} = \overset{\approx}{G}(x)$. Thus in this approach the gravitational constant may be not constant. Perhaps, the so called effect of dark matter can be explained with this argument.

5 Conclusions

Thus we have considered the effect of induced Riemann geometry in nonlinear electrodynamics. This effect can account for real gravitation.

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